Journées Calculabilités 2016

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A further introduction to forcing I,II

Grégory Lafitte & Laurent Bienvenu

The method of forcing was introduced by Paul Cohen in 1963 to prove the independence of the Continuum Hypothesis from the Zermelo-Fraenkel axioms of set theory. Following Boban Veličković's tutorial at the 2015 edition of the Journées Calculabilités, this tutorial will aim at giving a further introduction to the method of forcing. After recalling the fundamental notions introduced in the 2015 tutorial, the core of this two-part tutorial will be a detailed presentation of the various forcing notions, from the classical/historical forcing notions to applications in computability.

Continuous models of computation: from computability to complexity

Amaury Pouly

In 1941, Claude Shannon introduced a continuous-time analog model of computation, namely the General Purpose Analog Computer (GPAC). The GPAC is a physically feasible model in the sense that it can be implemented in practice through the use of analog electronics or mechanical devices. It can be proved that the functions computed by a GPAC are precisely the solutions of a special class of differential equations where the right-hand side is a polynomial. Analog computers have since been replaced by digital counterpart. Nevertheless, one can wonder how the GPAC could be compared to Turing machines.

A few years ago, it was shown that Turing-based paradigms and the GPAC have the same computational power. However, this result did not shed any light on what happens at a computational complexity level. In other words, analog computers do not make a difference about what can be computed; but maybe they could compute faster than a digital computer. A fundamental difficulty of continuous-time model is to define a proper notion of complexity. Indeed, a troubling problem is that many models exhibit the so-called Zeno's phenomenon, also known as space-time contraction.

In my thesis, I give several fundamental contributions to these questions. We show that the GPAC has the same computational power as the Turing machine, at the complexity level. We also provide as a side effect a purely analog, machine-independent characterization of P and Computable Analysis.

Computing & Programming with continuous dynamical systems

Olivier Bournez

In this talk, we will review some old and recent results concerning continuous dynamical systems. We will mainly focus on continuous time dynamical systems, i.e. computations by ordinary differential equations. Results will include universality results, as well as hardness results, with various senses. In particular, we will review results motivated by fields like computability/recursive analysis, but also computer algebra, and verification.

The busy beaver functions, Chaitin's number, and combinatorial properties of Kolmogorov complexity

Andrei Romashchenko

A binary string X is called stochastic if it is a typical element of some simple set A. Technically this means that the list of all elements of A can be printed by a short program, and Kolmogorov complexity of X is close to log of the cardinality of A. The definition of stochasticity is usually studied in the context of the classic busy beaver game and Chaitin's omega number: for every non stochasticity string its shortest description requires a very long time to converge; accordingly, evey non stochastic string has high mutual information with Chaitin's number. More recently it was discovered that (non)stochasticity has also interesting combinatorial implications. For example, the family of highly non stochastic strings enjoys the property of list decoding (Milovanov's theorem). On the other hand, the stochastic tuples of strings satisfy some natural criterion of extracting the mutual information.

Degrés Turing dans les sous-shifts minimaux

Pascal Vanier

Les sous-shifts sont des sous-ensembles de $A^{\mathbb{Z}^d}$ qui sont fermés invariants par translation, une autre manière de les définir est par le biais de motifs interdits. Les sous-shifts minimaux sont les sous-shifts ne contenant proprement aucun autre sous-shift. Ils sont fondamentaux dans le sens où tout sous-shift contient un sous-shift minimal. Nous nous intéressons ici à la structure des degrés Turing des sous-shifts minimaux : nous allons montrer que ceux-ci contiennent toujours des configurations de tous les degrés au dessus du degré de n'importe quel configuration, ils contiennent le cône de degrés au dessus de chacun de leurs degrés. Par ailleurs nous allons construire un sous-shift qui contient un nombre non dénombrable de cônes de bases disjointes.

Jeu de Tetris et génération de polyominos

Enrico Formenti

Dans cette exposé nous allons s'inspirer d'un mauvais joueur de Tetris pour construir un algorithme de génération de tous les polyominos préfixes de surface n dont la complexité en temps ammorti constant et espace $O(n)$.

Cellular Automata and Cyclic Codes

Luca Mariot

Cyclic codes represent one of the most studied classes of error-correcting codes, due to their rich algebraic structure. In fact, several types of linear codes such as BCH and Reed-Solomon codes fall into the broad category of cyclic codes.

In this talk, we show the equivalence between additive cellular automata (CA) and linear cyclic codes. In particular, we observe that preimage computation in additive CA corresponds to the systematic encoding procedure of cyclic codes through Linear Feedback Shift Registers (LFSR). On the other hand, syndrome computation in a cyclic code can be carried out in parallel by applying the corresponding CA global rule to a codeword. Further, we leverage on the theory of cryptographic boolean functions to characterize the minimum distance (and thus the error-correction capability) of cyclic codes induced by CA.

As an example, we finally present how to implement the (7,4,3) Hamming code using a radius 2 CA.

A Linear Acceleration Theorem for 2D Cellular Automata on all Complete Neighborhoods

Anaël Grandjean

Linear acceleration theorems are known for most computational models. Although such results have been proved for two-dimensional cellular automata working on specific neighborhoods, no general construction was known. We present here a technique of linear acceleration for all two-dimensional languages recognized by cellular automata working on complete neighborhoods.

Comportement séquentiel des schémas de récursion primitive

Pierre-Louis Curien

Nous analysons les schémas récursifs primitifs sous l'angle interactif fourni par la théorie des algorithmes séquentiels de Berry et Curien, et revisitons dans ce cadre un joli thèorème de Loïc Colson appelé théorème d'*ultime obstination* et qui dit que tout schéma récursif primitif finit toujours par appeler le même argument, excluant ainsi par exemple l'algorithme naturel de comparaison de deux entiers par décrémentation alternée des deux arguments. Le slogan général est que l'on ne s'intéresse pas seulement à ce qui est calculé, mais à comment le caldul est effectué. Cette étude fine n'est pas accessible à partir des modèles de calcul type machine de Turing, mais l'est à partir des modèles basés sur le lambda-calcul ou sur la récursion à la Kleene.

Computability and complexity in freezing cellular automata

Guillaume Theyssier

Un automate cellulaire est "freezing" si l'état d'une cellule ne peut que décroître au cours de l'évolution pour un ordre (partiel) fixé sur les états. Cette contrainte très forte appauvri beaucoup les dynamiques possibles, mais n'empêche pas d'encoder du calcul sous certaines formes. Cet exposé fera un tour d'horizon de la complexité des AC freezing en dimension 1 et 2 ainsi que de la décidabilité de certaines de leurs propriétés tant du point de vu topologique que mesuré.

A resolution of the Gamma question

Benoit Monin

The Gamma question was formulated by Andrews, Cai, Diamondstone, Jockusch and Lempp in "Asymptotic density, computable traceability and 1-randomness" (2013, available at `http://www.math.wisc.edu/~lempp/papers/traceable.pdf`). It is related to the recent notion of coarse computability which stems from complexity theory. The Gamma value of an oracle set measures to what extent each set computable with the oracle is approximable in the sense of density by a computable set. The closer to 1 this value is, the closer the oracle is to being computable. The above authors proved that some oracle have a Gamma value of 0 and that some oracle have a Gamma value of $\frac{1}{2}$. They also showed that an oracle have a Gamma value bigger than $\frac{1}{2}$ iff it has a Gamma value of 1, iff it is computable. The Gamma question asks whether this value can be strictly in between 0 and $\frac{1}{2}$. We answer the question by presenting a proof that an oracle have a Gamma value smaller than $\frac{1}{2}$ iff it has a Gamma value of 0, iff it computes a function which equals infinitely often every computable function bounded by 2^{2^n}.

Some ordinal time algorithmics

Sabrina Ouazzani

In this talk, we study the infinite time Turing machine (ITTM) computation model through the lens of algorithmic techniques. We explain how these algorithms enable computations to reach (rather) large ordinal halting times, and how we can use them to highlight some properties of the ITTM-computable ordinals.

Higher Randomness et caractérisation des hK-triviaux

Paul-Elliot Anglès D'auriac

La calculabilité, qui formalise la notion d'algorithme, nous a donné les outils nécessaires pour définir l'aléatoire. Ainsi, un ensemble sera considéré aléatoire s'il ne possède aucune propriété trop exceptionnelle, pour certaines classes de propriétés donnée par la calculabilité. Dans cet exposé, on s'interessera à une autre notion de calculabilité, définie par la théorie descriptive des ensembles mais que l'on peut voir comme des processus à temps ordinal. Cette nouvelle définition de calculable nous donne de la même manière de nouvelles notions d'aléatoire : l'aléatoire d'ordre supérieur. L'étude de ces notions, déjà interessante en elle-même, nous permet de par ses points communs et ses différences avec le cas classique de mieux le comprendre. Nous verrons ainsi une nouvelle caractérisation des K-triviaux d'ordre supérieur, qui n'a pas d'équivalent dans le cas classique, établie lors d'un stage avec Benoit Monin.

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